Reliability of tunnel liners

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Abstract

The possibility of application of structural reliability theory to the computation of the safety margins of excavated tunnels is presented. After a brief description of the existing procedures and the limitations of the safety coefficients such as they are usually defined, the proposed limit states are precised as well as the random variables and the applied methodology. Also presented are simple examples, some of them based in actual cases, and to end, some conclusions are established the most important one being the probability of using the method to solve the inverse problem of identification.

1 Foreword

In tunnel construction, as in every engineering work, it is usual the decision making with incomplete data. Nevertheless, consciously or not, the builder weighs the risks (even if this is done subjectively) so that he can offer a cost.

The objective of this paper is to recall the existence of a methodology to treat the uncertainties in the data so that it is possible to see their effect on the output of the computational model used and then to estimate the failure probability or the safety margin of a structure. In this scheme it is possible to include the subjective knowledge on the statistical properties of the random variables and, using a numerical model consistent with the degree of complexity appropriate to the problem at hand, to make rationally based decisions.

As will be shown with the method it is possible to quantify the relative importance of the random variables and, in addition, it can be used, under certain conditions, to solve the inverse problem. It is then a method very well suited both to the project and to the control phases of tunnel construction.

2 Modelling of perforation effects

Figures 1 and 2 show the classical approach to the understanding of the displacements observed during tunnel perforation. In an elastoplastic medium it is assumed that a plastic ring of radius ρ is formed. This ring grows consistently to the longitudinal transfer of load shown in figure 2.

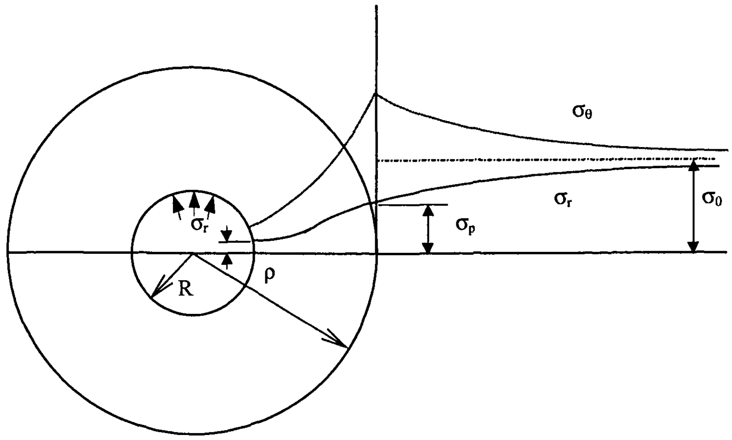


Figure 1.

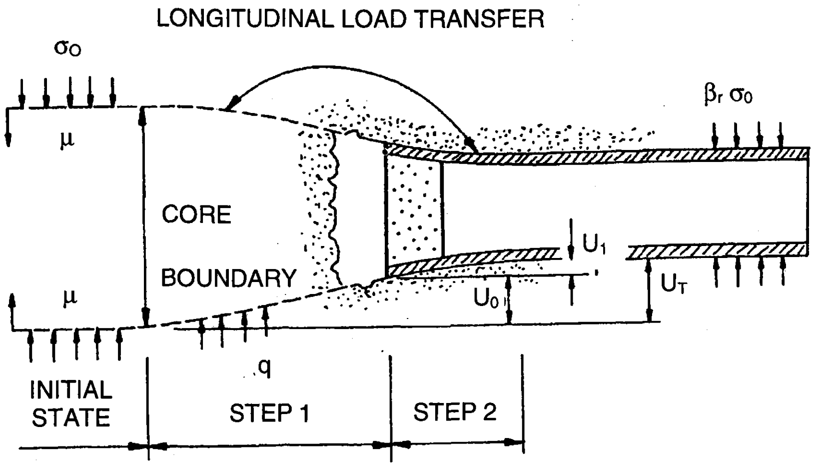


Figure 2.

In order to simplify the three-dimensional nature of the problem it is usual to treat the medium according to a three-dimensional simplification proposed by Panet [1] in which the internal pressure varies from the initial state to the final one (induced by the liner) according to a parameter λ whose meaning can be seen in figure 3.

This procedure, typical for circular tunnels can be extended to a general case using the finite element method as follows. First of all a rock mass is isolated in which an initial stress-state $\{\sigma^0\}$ as well as an overload p_0 on the upper border (equivalent to the geostatic pressure induced by the non-modelled part) are assumed. The remaining boundary is fixed to induce the initial stress-state generally defined using an at-rest coefficient K_0 and a vertical geostatic pressure σ_r as well as a plane-strain condition. This last one is not consistent with the general 3-

D randomness that should be rigorously assumed, but it is one of the prices to be paid in order to make the problem manageable in practical cases.

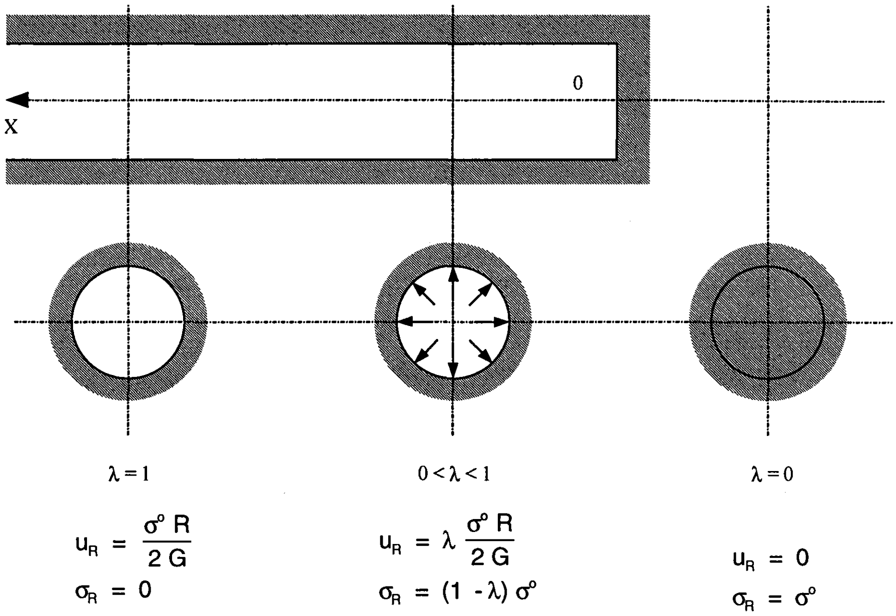


Figure 3.

Although generally the excavation process has different phases (for instance: top heading and bench) a case with total excavation and delayed application of the total liner is described in what follows, although this does not imply any limitation for the procedure that could be generalized to a greater number of steps.

If the void to be excavated is not represented in the mesh, the out-of-equilibrium forces {f} in the excavated boundary can be used to simulate the Panet procedure. Only a part λ\{f\} are applied before building the liner, as the 3-D effect is equivalent to the existence of a virtual liner able to sustain pressures (1-λ) {f}.

The forces equivalent to the unlined step are then

$$\{f_1\} = \lambda \{f\}. \tag{1}$$

and their effect can be obtained solving the problem

$$[K_1] \{u^1\} = \{f_1\}. \tag{2}$$

where $[K_1]$ is the stiffness matrix corresponding to the unlined mesh.

Stresses and displacements are then

$$\{\sigma\} = \{\sigma^1\} + \{\sigma^0\}$$
$$\{u\} = \{u^1\} \tag{3}$$

As soon as the concrete is projected, the front advances and the load transfer is completed

$$\{f_2\} = (1-\lambda)\ \{f\}. \tag{4}$$

Those loads can be automatically obtained if it is accepted that the initial stress state is now $\{\sigma^1\}+\{\sigma^0\}$. In addition in this step the liner is added so that a new stiffness matrix $[K_2]$ is assembled and the problem to be solved is

$$[K_2]\ \{u^2\} = \{f_2\}. \tag{5}$$

Solving those equations, stresses $\{\sigma^2\}$ and displacements $\{u^2\}$ are obtained. The last ones represent the "convergences" usually measured in-situ. It is possible to write then

$$\{u\} = \{u^1\} + \{u^2\}$$
$$\{\sigma\} = \{\sigma^1\} + \{\sigma^0\} + \{\sigma^2\} \tag{6}$$

The general scheme is represented in figure 4

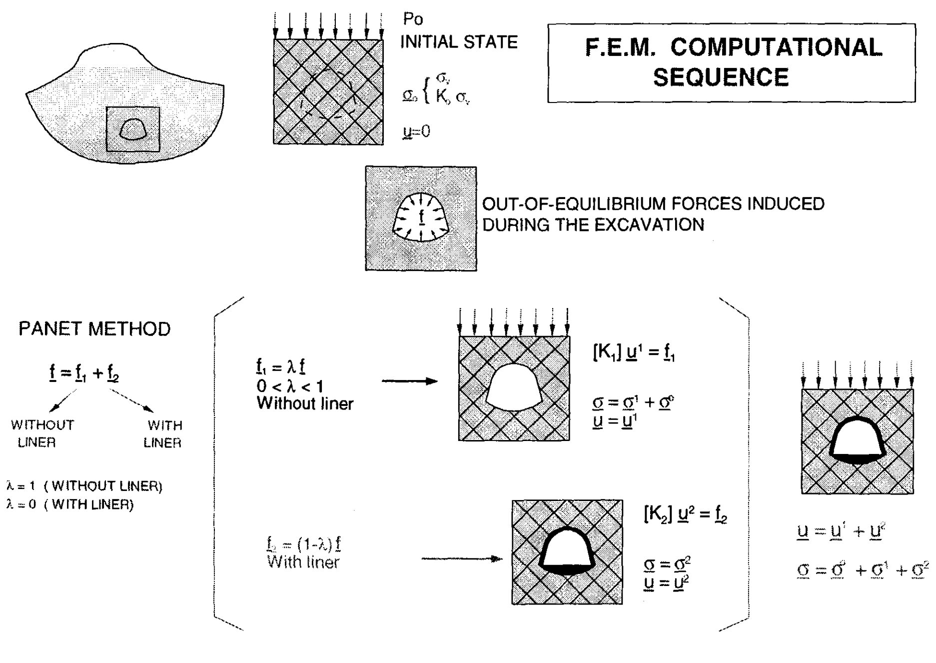


Figure 4.

3 Limit states

During the tunnel construction measures are taken systematically in relation with displacements (convergences, extensometers) and stresses (load cells) so it is natural to try to establish the limit states on those indexes. On the other hand there are two conditions that affect the builder, i.e.: the pay line and the need to assure the stability of the plastified soil ring.

In the first case the builder is obliged to follow the established geometry as closely as possible if he has to avoid an economic penalty but, in addition, an excessive soil displacement can act against him. In that sense the limitation of the relative displacements (convergences) can be considered as a service limit state. On the other hand the stability of the soil ring is also affected by the displacement because a limit has to be stabilised in order to avoid a catastrophic failure. In this sense the convergence would measure an ultimate limit state. To avoid that duplication and to control more direct factors it is desirable to analyze the stress state in some point of the mesh or, in order to have a more global representation, to use the plastification radius limiting it to a desired factor of the tunnel mean radius. The last criterion is generally used in design and seems very meaningful as an ultimate limit state, because it affects the stability directly.

Finally the liner safety has to be guaranteed. In this case it is possible to use the classical Rabcewickz [2] criteria that can be established using a condition on stresses.
In the example developed in what follows the limit states are:

a) The convergence has to be less than a limiting value
b) The "plastification" radius has to be less than a limiting value
c) The stress in the central fibber of the liner has to be less than a limiting value.

As a linear and elastic computational method is proposed, it is necessary to have some criteria to obtain a "plastification" radius. To do that the over-stress ratio (OSR) resulting after comparing the radius ϕ^* of the Mohr circle in a point with the maximum admissible one ϕ in a Mohr-Coulomb criteria is proposed, i.e.:

$$\mathrm{OSR} = \frac{\phi^*}{\phi} = \frac{|\sigma_1 - \sigma_3|}{2\cos\varphi + |(\sigma_1 + \sigma_3)|\sin\varphi} \quad . \tag{7}$$

where φ is the effective friction angle.

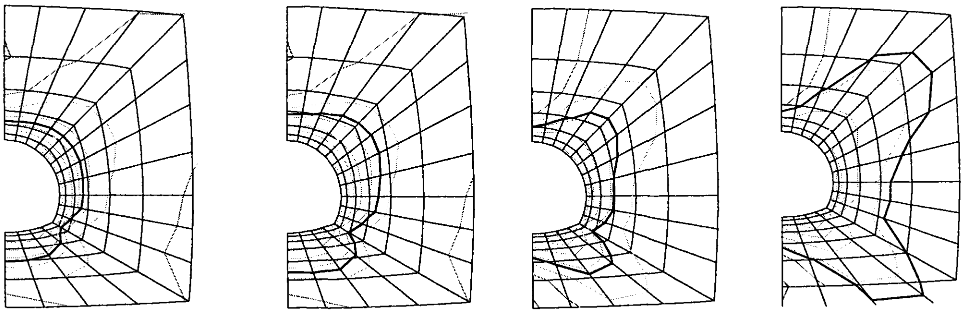
If, for instance, the limit state condition is

$$\rho_{\mathrm{plast}} \leq 2\ \mathrm{R}. \tag{8}$$

where R is the tunnel mean radius, it can be stated as

$$\mathrm{OSR} \leq 1 \text{ at a distance } 2\ \mathrm{R}. \tag{9}$$

A new problem is now that the OSR = 1 curve depends on the tunnel geometry and on the soil mechanical properties. Figure 5 shows some cases in which it is possible to see the existence of selected directions along which the limiting value is reached at different distances.



K=0.7, φ=30 , C=1. E5　　K=0.7, φ=22 , C=1. E5　　K=0.5, φ=30 , C=1. E5　　K=0.5, φ=22 , C=1. E5

Figure 5.

We propose that, after a study using mean values, the designer chooses the direction in which the curve OSR = 1 goes farther and in this way he can select the point in which the test (9) will be done.

4 Basic variables and level II methods

The randomness can be grouped in that related to the liner and the one collecting the soil properties. A common characteristic is the excavation geometry that affects both the stress distribution in the soil and the liner thickness. Although it is possible to introduce the geometry as a random variable, this study has been limited to consider only the randomness of the liner thickness.
Among the variables considered are the following.

In relation to material properties:

- In the soil: Young modulus, density, cohesion, initial stress state, etc
- In the liner: thickness, density, etc

In relation with the simulation procedure:

- Panet coefficient, excavation steps (heading, bench), etc

Generally the variables are not independent and, in addition to the mean values their definition includes the correlation matrix.

In addition, soil properties have to be defined as random fields (Journel [3]; Magnam [4]; Van Marcke [5]; Li [6]). In most actual occasions the difficulty of disposal of sufficient data produces the simplification of considering total correlation inside different zones. It is for instance usual to define a ring around the tunnel in which a Young modulus lesser than that used in the rest is used in order to model in a simplified manner the soil plastification.

To establish the reliability a First Order Second Moment method is used in which (figure 6) the basic variables {z} are transformed to a standard normal space {y} in which the problem is reduced (Madsen [7]) to compute the point $\{y^*\}$ in the failure surface corresponding to the limit state under study g(z) such that it minimises

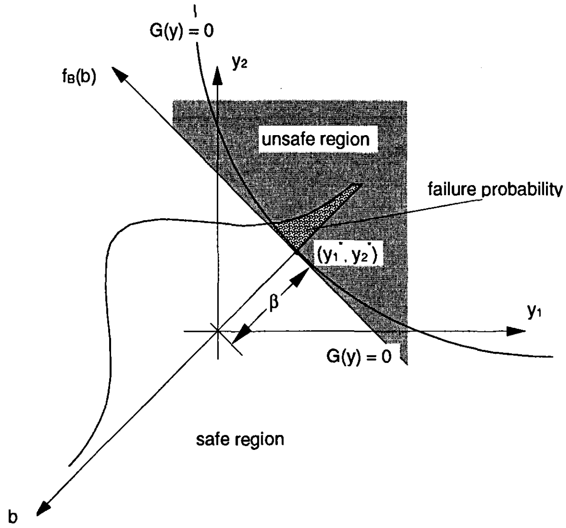


Figure 6.

$$Q = \frac{1}{2}\{y\}^T\{y\}. \tag{10}$$

under the condition

$$G(\{y^*\}) = 0; \quad G(\{y\}) = G([T]\{z\}) = g(z) = 0. \tag{11}$$

The obtained point is called the **design point** and can be written as

$$\{y^*\} = \beta\ \{\alpha\}. \tag{12}$$

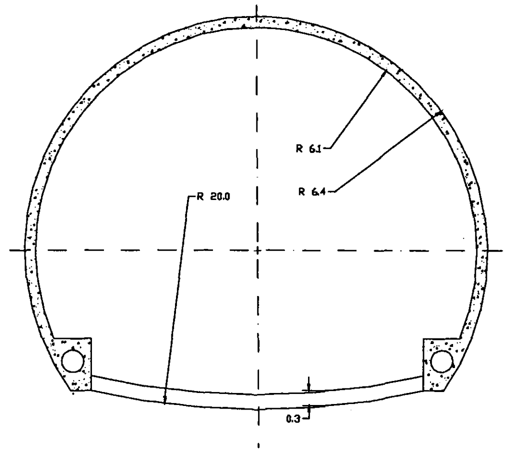
where β is called **reliability index** and $\{\alpha\}$ are the sensitivities of the design point with respect to each basic variable.

The algorithm to search the minimum of eq. 10 & 11 is characteristic of every method. In our case the well-known Hasofer-Lind-Rackwitz-Fiessler algorithm (Madsen [7]) has been used. The iterations are done using the Taylor development of the finite element equations around the current design point according to the well known philosophy of Probabilistic Finite Elements (Kleiber [8]).

Finally it is interesting to point out that this method can be used to solve the inverse problem if only a unique condition has to be adjusted (for instance the convergence or the stress measured at a point). It is only necessary to define the measured value as a new limit state and to search the point $\{y^*\}$ corresponding to the design point of that state.

5 Example

A road tunnel whose general dimensions can be seen in figure 7 has been chosen to show the performance of the method. Also the mean material properties are shown.



Liner

Density $\rho = 2500$ kg/m^3
Young modulus $E_S = 1.5 \cdot 10^{10}$ N/m^2
Poisson coefficient $\nu = 0.25$

Soil

Density $\rho = 2300$ kg/m^3
Young modulus $E_{15} = E_{16} = 4 \cdot 10^9$ N/m^2
Poisson coefficient $\nu = 0.18$
Overload $p_0 = 6.47 \cdot 10^6$ N/m
Cohesion $C = 2 \cdot 10^5$ N/m^2
Friction angle $\varphi = 25°$
At-rest coefficient $K_0 = 1$
Panet coefficient $\lambda = 0.7$

Figure 7.

The tunnel is 330 m under the surface and the model only takes 43 m above the key, which implies that an overload $p_o = \rho\ g\ H = 2300 \cdot 9.8 \cdot 287 = 6.47 \cdot 10^6$ N/m has to be applied in the upper boundary. Both the soil and the liner have been modelled using 8 nodes isoparametric elements. Figure 8 shows the mesh in which the simetry has been accepted and also the different materials that have been used: 14 for the liner and 2 for the soil. The same figure shows the position of the nodes referred to in the limit-state definitions.

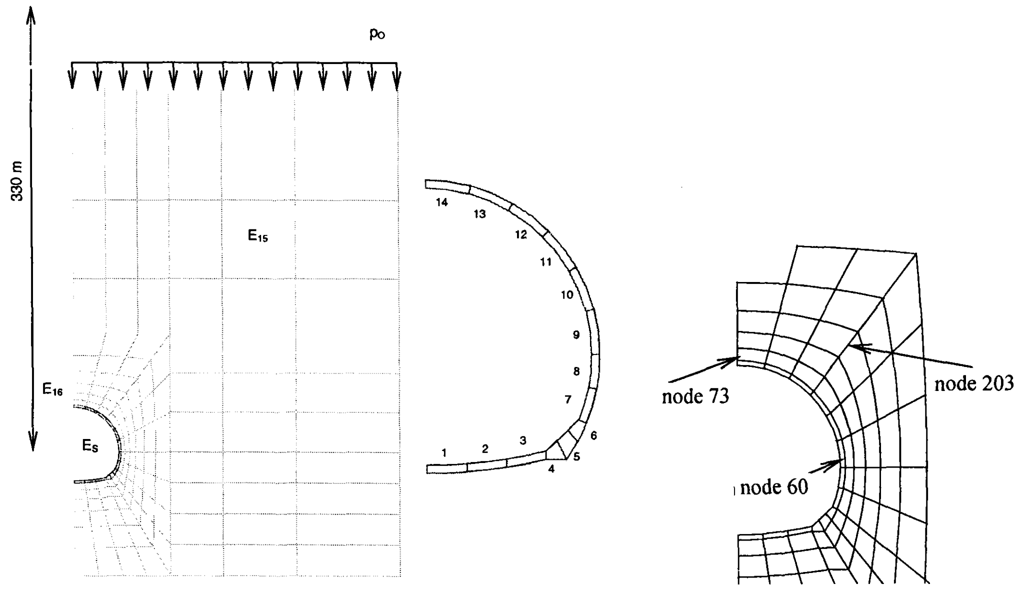


Figure 8.

In this example the random variables are the following:

- Soil Young modulus (E_{15}, E_{16})
- Liner Young modulus (E_1, E_2, ..., E_{14})

- Overload p_0
- At-rest coefficient K_0
- Panet coefficient λ

The distributions assumed for Young moduli are log-normal while K_0 and λ are considered as beta distributed, bounded at both sides, p_0 is assumed also log-normal. The definition of some of those functions is contained in figure 9, while the variation coefficients and the correlation matrix are shown in figure 10.

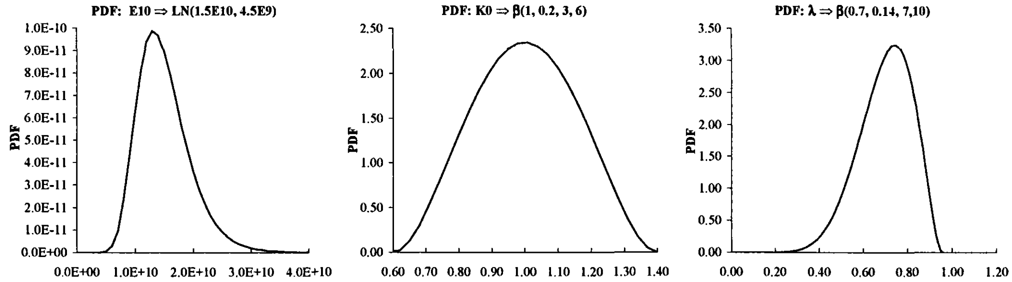


Figure 9.

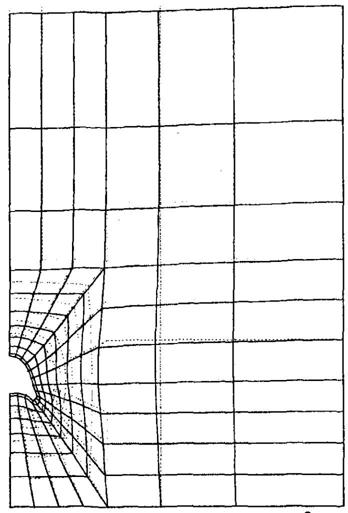
	PDF	V.C.	K_o	λ	p_o	E_{15}	E_{16}	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9	E_{10}	E_{11}	E_{12}	E_{13}	E_{14}
K_o	BETA	20	100																		
λ	BETA	20		100																	
p_o	LN	10			100																
E_{15}	LN	40				100								0							
E_{16}	LN	40					100														
E_1	LN	10						100		90											
E_2	LN	10							100												
E_3	LN	10								100											
E_4	LN	20									100										
E_5	LN	20										100			80						
E_6	LN	20											100						10		
E_7	LN	20												100							
E_8	LN	20													100						
E_9	LN	20														100					
E_{10}	LN	30						Symmetric									100				
E_{11}	LN	30																100		80	
E_{12}	LN	30																	100		
E_{13}	LN	30																		100	
E_{14}	LN	30																			100

Figure 10.

The limit states are defined as follows:

a) **Serviceability limit state:** Maximum displacement is defined as R/100, i.e.: U_{lim} (node 73) < R/100 ≈ 6.4 cm. The results are presented in figure 11.

Variable	Design Point Physic Value	Sensitivity Coefficient	Weighting Coefficient
E_1	1.4916E+10	0.14290E-02	9.94376E-01
E_2	1.4916E+10	0.33941E-03	9.94374E-01
E_3	1.4915E+10	0 .25722E-03	9.94364E-01
E_4	1.4680E+10	0.20897E-02	9.78693E-01
E_5	1.4681E+10	0.65874E-03	9.78713E-01
E_6	1.4681E+10	0.42718E-03	9.78704E-01
E_7	1.4680E+10	0.39637E-03	9.78665E-01
E_8	1.4683E+10	-0.13265E-03	9.78864E-01
E_9	1.4683E+10	-0.11042E-03	9.78864E-01
E_{10}	1.4244E+10	0.60922E-02	9.49620E-01
E_{11}	1.4243E+10	0.21744E-02	9.49519E-01
E_{12}	1.4242E+10	0.14456E-02	9.49451E-01
E_{13}	1.4241E+10	0.11239E-02	9.49403E-01
E_{14}	1.4241E+10	0.92319E-03	9.49378E-01
E_{15}	0.1559E+10	0 .67488	3.89616E-01
E_{16}	0.1558E+10	0.22585	3.89454E-01
λ	0.90521	-0.35512	1.29316E+00
K_0	1.3427	-0.39665	1.34267E+00
p_0	0.9576E+07	-0.45830	1.48087E+00

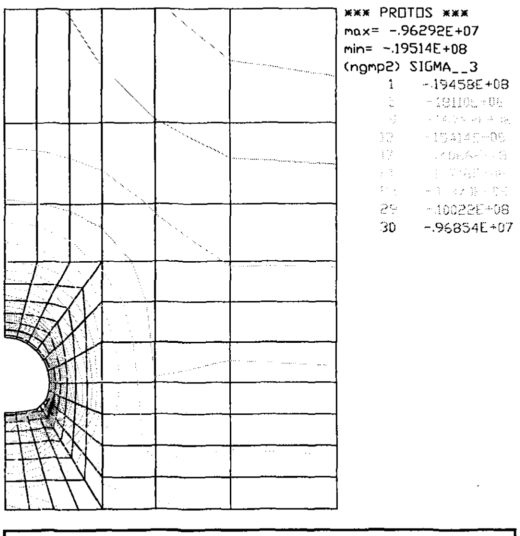


β = 4.5275; P_f = 0.3034·10^{-3} %

Figure 11.

b) **Ultimate limit state 1 (liner):** It is considered that the liner fails when the stress reaches $150{\cdot}10^5$ N/m^2, i.e.: σ_{lim} (node 60) < $150{\cdot}10^5$ N/m^2. (Figure 12).

Variable	Design Point Physic Value	Sensitivity Coefficient	Weighting Coefficient
E_1	1.4902E+10	0.11482E-02	9.93496E-01
E_2	1.4902E+10	0.13811E-02	9.93488E-01
E_3	1.4902E+10	0.23172E-02	9.93456E-01
E_4	1.5320E+10	0.23494E-02	1.02132E+00
E_5	1.5321E+10	0.20072E-02	1.02137E+00
E_6	1.5321E+10	0.19822E-02	1.02137E+00
E_7	1.5320E+10	0.20363E-02	1.02136E+00
E_8	1.5328E+10	-0.13418E-02	1.02184E+00
E_9	1.5341E+10	-0.78791E-02	1.02276E+00
E_{10}	2.3364E+10	-0.20494E-01	1.55762E+00
E_{11}	2.3461E+10	-0.40663E-01	1.56406E+00
E_{12}	2.3616E+10	-0.72850E-01	1.57438E+00
E_{13}	2.3931E+10	-0.13781	1.59543E+00
E_{14}	2.4827E+10	-0.31766	1.65516E+00
E_{15}	0.3127E+10	0.19889	7.81721E-01
E_{16}	0.2604E+10	0.37794	6.50957E-01
λ	0.81296	-0.55355	1.16138E+00
K_0	1.1145	-0.30786	1.11451E+00
p_0	0.9187E+07	-0.53762	1.42069E+00

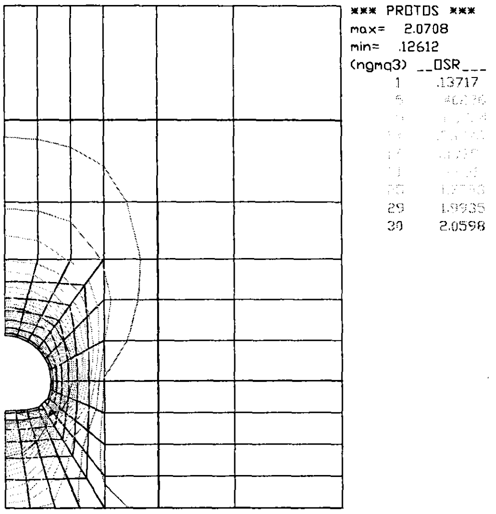


β = 3.7754; P_f = 0.8001·10^{-2} %

Figure 12.

c) **Ultimate limit state 2 (soil)** The OSR parameter limited to 1 at a distance 1.5 R, i.e.: OSR (node 203) < 1. The results are presented in figure 13.

Variable	Design Point Physic Value	Sensitivity Coefficient	Weighting Coefficient
E_1	1.4926E+10	0.16813E-03	9.94974E-01
E_2	1.4926E+10	0.60879E-04	9.94970E-01
E_3	1.4926E+10	0.17146E-03	9.94951E-01
E_4	1.4181E+10	0.44568E-01	9.48205E-01
E_5	1.4179E+10	0.15132E-01	9.48087E-01
E_6	1.4176E+10	0.98665E-02	9.47902E-01
E_7	1.4163E+10	0.93342E-02	9.47114E-01
E_8	1.4159E+10	0.78475E-02	9.46935E-01
E_9	1.4139E+10	0.97682E-02	9.45787E-01
E_{10}	1.2994E+10	0.72234E-01	8.78582E-01
E_{11}	1.3175E+10	0.51478E-02	8.89903E-01
E_{12}	1.3248E+10	-0.28830E-02	8.93159E-01
E_{13}	1.2964E+10	0.32281E-01	8.75771E-01
E_{14}	1.2962E+10	0.24778E-01	8.76076E-01
E_{15}	0.3833E+10	0.13177E-01	9.43845E-01
E_{16}	0.4616E+10	-0.16240	1.14858E+00
λ	0.83309	-0.87686	1.20440E+00
K_0	1.2167	-0.41074	1.15819E+00
p_0	0.7323E+07	-0.16249	1.10846E+00



$\beta = 3.8047$; $P_f = 0.7112 \cdot 10^{-2}$ %

Figure 13.

As can be seen, for the 3 states results obtained for the reliability indexes are high so that the failure probability is low.

The most influential variables (shaded data in figures 11 to 13) are the soil Young modulus, the at-rest coefficient, the Panet one and the overload. The liner Young moduli are not much important except for the limit state b) where it is possible to see the relative importance of the elements near to the key and how important is the correlation among the variables defining the liner vault (moduli 10 to 14). In this way the temptation to define material 10 as deterministic has to be avoided due to the correlation with the highly influential material 14. It can be seen that the weighting coefficients of both materials are similar.

In all cases the values of the at-rest coefficient, Panet coefficient and overload at the design points are higher than the mean values (weighting coefficient higher than 1) indicating that they are "load" variables. On the contrary, the soil Young modulus is a "resistance" variable for the limit states a) and b) while it is a "load" one for the OSR limit state.

Finally in order to show the possibilities of the method for the **inverse problem** let us imagine now that the instrumentation has provided data that we should like to use to establish the stress state in the system. In particular, imagine that a loading cell at the key point has registered a normal stress of $70 \cdot 10^5$ and that the horizontal convergence at the vault springs is 4 cm.

If the design point is obtained for every condition considered as a limit state we shall have the values of the basic variables giving the maximum failure probability, i.e.: we obtain the worst combination consistent with the registered measurement. Obviously the procedure **does not solve simultaneously** the problem for both conditions (a system approach should be pertined) but it is possible to use a sequential procedure trying to accomplish every condition and, if necessary to use a combination of the results. This is what we have done in the present example as shown in figure 14.

Analysis Type	conv mm	σ_{liner} (10^5) N/m²	point	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9	E_{10}	E_{11}	E_{12}	E_{13}	E_{14}	E_{15}	E_{16}	K_0	λ	p_0
				(10^{10}) N/m²																Adim	Adim	(10^6) N/m
Determ.	2.21	80.61	initial val	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	4.0	4.0	1.0	0.7	6.47
Probab.	“	“	initial val	“	“	“	“	“	“	“	“	“	“	“	“	“	“	“	“	“	“	“
	4.00	106.5	Design p 1	1.49	1.49	1.49	1.47	1.47	1.47	1.47	1.47	1.47	1.42	1.42	1.42	1.42	1.42	3.05	2.93	1.09	0.75	7.78
Probab.	“	“	initial val	“	“	“	“	“	“	“	“	“	“	“	“	“	“	“	“	“	“	“
	2.52	70.00	Design p 2	1.49	1.49	1.49	1.41	1.41	1.41	1.41	1.41	1.41	1.00	1.00	0.99	0.99	0.95	3.25	3.65	1.00	0.65	6.47
Probab.	“	“	initial val	“	“	“	“	“	“	“	“	“	“	“	“	“	“	“	“	“	“	“
	4.00	90.29	Design p 3	1.48	1.48	1.48	1.38	1.38	1.38	1.38	1.38	1.38	0.09	0.09	0.09	0.09	0.09	2.62	2.86	1.07	0.69	7.40
Probab.	“	“	initial val	“	“	“	“	“	“	“	“	“	“	“	“	“	“	“	“	“	“	“
	3.34	70.00	Design p 4	1.47	1.47	1.47	1.34	1.34	1.34	1.34	1.34	1.34	0.08	0.08	0.08	0.08	0.07	2.62	3.06	1.03	0.66	6.77
Probab.	“	“	initial val	“	“	“	“	“	“	“	“	“	“	“	“	“	“	“	“	“	“	“
	4.00	74.31	Design p 5	1.46	1.46	1.46	1.31	1.31	1.31	1.31	1.31	1.31	0.08	0.08	0.08	0.07	0.07	2.37	2.75	1.06	0.68	6.98
Probab.	“	“	initial val	“	“	“	“	“	“	“	“	“	“	“	“	“	“	“	“	“	“	“
	4.02	70.00	Design p 6	1.46	1.46	1.46	1.28	1.28	1.28	1.28	1.28	1.28	0.07	0.07	0.07	0.07	0.07	2.28	2.66	1.06	0.68	6.80

Measured or fixed values

Computed Values

Figure 14.

The figures inside shaded areas are such that either have been measured or fixed during the procedure. The first step is to compute using a limit state on the convergence value. In this way a stress figure of $106 \cdot 10^5$ N/m² has been obtained which is different of the actual one. Then using the values of the design point a limit stress state is fixed which after solving produces a convergence value of 2.52 cm. The procedure continues until an admissible combination is obtained.

6 Conclusions

In this paper we have shown the possibility of combining the ideas of probabilistic finite element methods with the computation of tunnels reliability using Panet method. The idea is to have a procedure to quantify the randomness transmitted to the results by that contained in the data usually accompanying the project of construction of tunnels in real life.

With the described methods it is possible to analyze the impact of assumed incertitude of the data on the accomplishment of certain limit states. The ones described here are common practice in tunnel projects but its is possible to imagine others for special cases and which implementation does not offer any special difficulty.

An interesting by-product of the method is the computation of the sensitivity of the result to different variables, what allows to consider deterministic those less influential and to increase the tests on those more important in order to reduce the incertitude. There is also a direct relationship among those sensitivity coefficients and the weighting factors typical of structural analysis. After a repeated use of method they could allow the preparation of recommendations for structural tunnel analysis.

Finally, the in situ measurements can be used to calibrate the model so that it is possible to get an idea of the values taken by other variables of interest as the soil-structure interaction pressure, the size of the soil ring co-operating whit the liner, etc.

In addition it is possible to use the measured data for the inverse problem, between certain limits that can be improved using the reliability of systems theory.

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